Ch 14: Partial Derivatives, Tangent Lines, and Tangent Planes

MATH 233 Summer Session I 2017

Worksheet (to be turned in)

In this exercise we will consider the function of two variables

$$f(x, y) = \frac{x^2}{4} + \frac{y^2}{16}$$

1 Find and sketch the level curves of the graph of $z = f(x, y)$. Find and sketch the vertical traces of the graph (try a couple values of k for both x and y). Use this information to sketch the graph. You will be adding to the sketch of your graph, so make it large enough to do so.

2 Find $f_x(x, y)$ and $f_y(x, y)$.

3 Consider the point on your surface $P(2, 4, 2)$. Verify this is on your surface. Plot it on your sketch. Evaluate f_x and f_y at this point. What do these values represent geometrically?

4

(a) Find the trace of the surface intersecting with the plane $y = 4$. Note that P lies in this intersection. Sketch this curve on your graph.

(b) If we restrict to this plane, we have a curve in a 2-dimensional coordinate system. What is the equation for the tanget line to the curve at the point corresponding to P? (in the 2-D coordinate system)

(c) Sketch the line from part b on your graph. Note that this line lies in the plane $y = 4$.

(d) How does the derivative of the 2-D curve at P compare to the partial derivative f_x at P?

5 Find a vector in 3 dimensional space that points in the direction of the tangent line you found in 4. (*hint:* To do this, suppose you start at P then move 1 in the positive x direction. How much do you have to move in the y and z directions to get to a point on your line?)

6

(a) Find the trace of the surface intersecting with the plane $x = 2$. Note that P lies in this intersection. Sketch this curve on your graph.

(b) If we restrict to this plane, we have a curve in a 2-dimensional coordinate system. What is the equation for the tanget line to the curve at the point corresponding to P? (In the 2-D coordinate system)

(c) Sketch the line from part b on your graph. Note that it lies in the plane $x = 2$.

(d) How does the derivative of the 2-D curve at P compare to the partial derivative f_y at P?

7 Find a vector that points in the direction of the tangent line you found in 6.

8 Use your answers above to find an equation for the plane passing through $P(2, 4, 2)$ containing both tangent lines. Note that this is an equation for the tangent plane to the surface at $P(2, 4, 2)$.